Classical Electrodynamics Quiz Ch 3 2012/04/23

The *Dirichlet* Green function for the region *inside* of a sphere of radius b was found to be

$$G_{\mathrm{D}}(\mathbf{x},\mathbf{x}') = \sum_{l=0}^{l=\infty}\sum_{m=-l}^{m=+l} \frac{4\pi}{2l+1} r_<^l \left(\frac{1}{r_>^{l+1}} - \frac{r_>^l}{b^{2l+1}}\right) Y_{lm}^*(\theta',\phi') Y_{lm}(\theta,\phi).$$

For the same region you are asked to give the *Neumann* Green function.

Keep in mind that whereas the Dirichlet Green function satisfies vanishing boundary conditions, the Neumann Green function satisfies instead the boundary condition

$$\frac{\partial G_{\mathrm{N}}}{\partial n'}|_{\mathbf{x}'\in S} = -\frac{4\pi}{S}.$$

Classical Electrodynamics Quiz Ch 4 2012/05/07

Consider a linear homogeneous dielectric material with constant ϵ_1 and a *uniform* applied electric field E_0. Now suppose one places in it an infinitely long *cylindrical rod* of radius a made of a linear homogeneous dielectric material with constant ϵ_2. Using *cylindrical coordinates* (ρ, ϕ, z) with the rod in the z direction and the applied field in the $\phi = 0$ direction, please find the *scalar potential* both inside and outside the rod.

Hint: Because everything is uniform in the z direction, the problem is effectively a 2D plane polar coordinate problem.

Classical Electrodynamics Quiz Ch 5 2012/05/21

Recall that quite generally $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$.

A *hard ferromagnet* can be modeled by assuming $\mathbf{M}$ is independent of $\mathbf{B}$ and $\mathbf{H}$. For vanishing $\mathbf{J}$, since $\nabla \times \mathbf{H} = 0$, one can take then $\mathbf{H} = -\nabla\Phi_{\mathrm{M}}$. Several integral formulas for the solution of $\nabla^2\Phi_{\mathrm{M}} = \nabla\cdot\mathbf{M}$ were found, including

$$\Phi_{\mathrm{M}} = -\frac{1}{4\pi}\nabla\cdot\int \frac{\mathbf{M}(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} d^3x'. \tag{1}$$

Please evaluate the above integral for *a uniformly magnetized sphere* of radius a, i.e., $\mathbf{M} = M_0\hat{\mathbf{e}}_3$, for $r' \le a$. [*Hint*: use $|\mathbf{x}-\mathbf{x}'|^{-1} = \sum \frac{r_<^l}{r_>^{l+1}} P_l(\cos\gamma)$]